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$$\begin{aligned}
& + \left(1 - \frac{(n-1)^2}{n^2}\right) \Big] \\
& = \frac{\pi r^3}{n} \left[n - \left(\frac{1}{n}\right)^2 - \left(\frac{2}{n}\right)^2 - \left(\frac{3}{n}\right)^2 - \dots - \left(\frac{n-1}{n}\right)^2 \right] \\
& = \pi r^3 \left\{ 1 - \frac{1}{n^3} [1^2 + 2^2 + 3^2 + \dots + (n-1)^2] \right\}.
\end{aligned}$$

Now the series $1^2 + 2^2 + 3^2 + \dots + (n-1)^2$ is the sum of the squares of the first $n-1$ positive integers. The formula for the sum of the squares of the first n integers is given in any College Algebra under "piles of shot" and is

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Hence

$$1^2 + 2^2 + 3^2 + \dots + (n-1)^2 = \frac{(n-1)n(2n-1)}{6}.$$

Substituting this in the expression for V_2 we have

$$\begin{aligned}
V_2 &= \pi r^3 \left[1 - \frac{1}{n^3} \cdot \frac{(n-1)n(2n-1)}{6} \right] \\
V_2 &= \pi r^3 \left[1 - \frac{1}{n} \left(1 - \frac{1}{n}\right) \cdot \frac{1}{2} \left(2 - \frac{1}{n}\right) \right].
\end{aligned}$$

Now proceed to the limit and we have

$$V = \lim_{n \rightarrow \infty} V_2 = \pi r^3 \left[1 - \frac{1}{2} (1) \cdot \frac{1}{2} (2) \right] = \pi r^3 \left[1 - \frac{1}{2} \right] = \frac{1}{2} \pi r^3$$

which is the well-known result.

DERIVATION OF FORMULA FOR $\tan \frac{1}{2} A$ IN SPHERICAL TRIGONOMETRY.

By GEORGE E. DEAN, School of Mines, Rolla, Mo.

Applying Napier's Rules of Circular Parts to the triangle formed by the bisector of angle A , the radius of the inscribed circle and one of the sides, we see that $\sin(s-a) = \tan r \cot \frac{1}{2} A$. Hence, $\tan \frac{1}{2} A = \tan r / \sin(s-a)$, so that

$$\frac{\tan \frac{1}{2}A}{\tan \frac{1}{2}B} = \frac{\sin(s-b)}{\sin(s-a)} \dots (1).$$

The sine proportion gives

$$\frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}(a+b)}{\tan \frac{1}{2}(a-b)} \dots (2).$$

For convenience, put $\tan \frac{1}{2}A = x$, $\tan \frac{1}{2}B = y$, $\tan \frac{1}{2}(a+b) = m$, $\tan \frac{1}{2}(a-b) = n$, $\sin(s-b) = p$, $\sin(s-a) = q$. Then

$$\frac{\frac{x+y}{1-xy}}{\frac{x-y}{1+xy}} = \frac{m}{n} \dots (3),$$

$$\text{and } \frac{x}{y} = \frac{p}{q} \dots (4).$$

$$\text{From (3), } \frac{1+xy}{1-xy} = \frac{x-y}{x+y} \cdot \frac{m}{n}. \quad \text{From (4), } \frac{x-y}{x+y} = \frac{p-q}{p+q}.$$

$$\text{Therefore, } \frac{1+xy}{1-xy} = \frac{p-q}{p+q} \cdot \frac{m}{n} = \frac{\tan \frac{1}{2}(a+b)}{\tan \frac{1}{2}c} \dots (5).$$

$$xy = \frac{\tan \frac{1}{2}(a+b) - \tan \frac{1}{2}c}{\tan \frac{1}{2}(a+b) + \tan \frac{1}{2}c} = \frac{\sin(s-c)}{\sin s} \dots (6).$$

Multiplying (4) by (6),

$$x^2 = \frac{\sin(s-b) \sin(s-c)}{\sin s \sin(s-a)} = \tan^2 \frac{1}{2}A.$$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

CRITICISM ON SOLUTION OF PROBLEM 163, DECEMBER NUMBER.

BY J. M. ARNOLD, CROMPTON, R. I.

The published solution is correct as far as the algebraical part is concerned but when it comes to pairing the couples there is quite a mixing up of the names.